ADAPTIVE INTERFEROMETRIC METHOD FOR INITIAL INTERFERENCE ORDER MEASUREMENT IN THE IMAGE OF HIGHLY ORIENTED POLYMERIC TEXTILE FIBERS

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Quite simple interferometric method based on the multi-fringe optical Fourier transform technique is suggested in this article. This method offers a specific and precise technique for the study of highly oriented fibers that produce two or more fringes in the marginal zone of the exit pupil of the microscope objective. The measuring accuracy of the direct initial interference order and the current interference orders in the image of highly oriented polymeric textile fibers is increased.

INTRODUCTION

Two versions of the variable wavelength optical Fourier transforms were described in previous articles [1–5]. These versions are single- and multi-fringe optical Fourier transform techniques. The original variable wavelength optical Fourier transform technique was studied by Pluta [6–8]. In this conventional technique, the intensity analysis method is applied. The intensity of the annular dark fringe center can be

consecutively changed from maximally bright to dark when the wavelength of the continuous monochromatic light varied from the short to the long region of visible spectrum or vice versa. The initial interference order, the current interference orders in the object image, and the spectral dispersion of the object birefringence are manually measured.

Using single- and multi-fringe techniques, the automatic intensity analysis method at the interference pattern center is applied to measure only the initial interference order and a new extended mathematical model is employed to determine the interference orders and the spectral dispersion of the object birefringence [1−4]. Sometimes the optical Fourier transforms patterns are noisy images, this is the problem that faced these techniques. Sadik and Litwin [5] applied an extended mathematical model and an extrapolation method that is based on the single-fringe variable-wavelength interferometry optical Fourier transform technique for object birefringence measurement. This method is used to overcome the problem of the noisy image at the interference pattern center. Its measuring accuracy of initial interference order and current interference orders measurements in the birefringent objects is increased compared with the previous method [3]. The advantage of this method is that it is not a time-consuming method if the object is weakly and/or moderately birefringent (one annular dark fringe appears in the marginal zone of the exit pupil of the microscope objective) are investigated. Otherwise, it is a time-consuming method if the highly birefringent object is examined (two or more fringes appear in the marginal zone of the exit pupil of the microscope objective). Additionally, the dispersion of both the microscope objective focal length and the difference between the ordinary and extraordinary focal lengths of the highly oriented object is measured [5]. This means that the measuring error is increased when the highly birefringent object is investigated.

For these reasons, an interferometric method based on the multifringe variable-wavelength interferometry optical Fourier transform technique is suggested. This method is applied for direct measurement of the initial interference order and the interference orders in the image of the highly birefringent objects. The advantage of this method is that the radii of the annular dark fringes are only measured at different wavelengths in visible spectrum and the measuring error is decreased.

THEORY

When a highly birefringent object of thickness t is investigated two or more fringes may appear in the back focal plane of the microscope

objective (Fourier Plane) as shown in Figure 1. Assuming that there are n annular dark fringes of radii $r_1, r_2, r_3, \ldots, r_n$ appearing in the exit pupil of the microscope objective when a monochromatic light of wavelength λ_1 is used, the values of the radii of the annular dark fringes can be written as follows:

$$r_1^2 = \left(\frac{2f_{ob}^2}{\Delta f}\right)_{\lambda_1} (m_1 \lambda_1 - t\mathrm{B}_1), \tag{1}$$

$$r_2^2 = \left(\frac{2f_{ob}^2}{\Delta f}\right)_{\lambda_1} ((m_1 + q_1)\lambda_1 - t\mathrm{B}_1), \tag{2}$$

$$r_3^2 = \left(\frac{2f_{ob}^2}{\Delta f}\right)_{\lambda_1} ((m_1 + q_2)\lambda_1 - t\mathrm{B}_1), \tag{3}$$

For nth fringe, the radius (r_n) is given by:

$$r_n^2 = \left(\frac{2f_{ob}^2}{\Delta f}\right)_{\lambda_1} ((m_1 + q_{n-1})\lambda_1 - t\mathrm{B}_1) \tag{4}$$
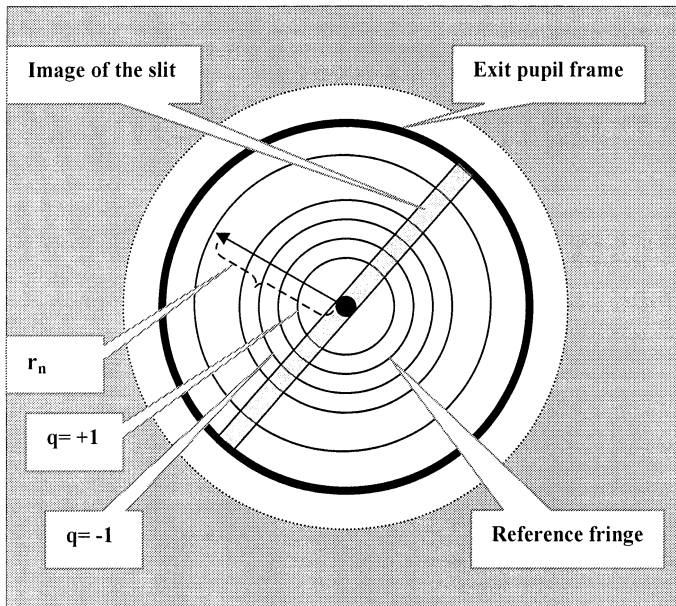

FIGURE 1 Illustration of the optical Fourier transform pattern in the back focal plane of the microscope objective.

where B is the object birefringence, f_{ob} the objective focal length, $\Delta f = f_\perp - f_\parallel$; $f_\perp$ and $f_\parallel$ are the ordinary and extraordinary focal lengths of the object, respectively, and $q_j = \pm j$ ($j = 1, 2, 3, \ldots, n - 1$) is the integer increment or decrement of the interference order at r_n with respect to r_1. $q = +j$ if $r_1 > r_2 > r_3 \cdots > r_n$ and $q = -j$ if $r_n > r_{n-1} > r_{n-2} \cdots > r_1$.

Using Eqs. 4 and 1, the following equation can be obtained

$$\left(\frac{r_1}{r_n}\right)^2_{\lambda_1} = \frac{m_1\lambda_1 - tB_1}{(m_1 + q_{n-1})\lambda_1 - tB_1} \tag{5}$$

Let $\sigma_1 = \left(\frac{r_1}{r_n}\right)^2_{\lambda_1}$, we get

$$tB_1 = \frac{m_1\lambda_1(1 - \sigma_1) - \sigma_1 q_{n-1}\lambda_1}{(1 - \sigma_1)} \tag{6}$$

Let the light wavelength change from λ_1 to λ_2, Eq. 6 becomes

$$tB_2 = \frac{m_2\lambda_2(1 - \sigma_2) - \sigma_2 q_{n-1}\lambda_2}{(1 - \sigma_2)} \tag{7}$$

where, $\sigma_2 = \left(\frac{r_1}{r_n}\right)^2_{\lambda_2}$, $m_2 (= m_1 + \kappa_1)$ is the current interference order and $\kappa_1 = \frac{(r_n)_{\lambda_2} - (r_n)_{\lambda_1}}{2(r_n)_{\lambda_2}}$ is the digital increment or decrement of the interference order m_2 with respect to m_1.

In general, Eq. 7 can be written in the form:

$$tB_s = \frac{(m_1 + i + \kappa_s)\lambda_s(1 - \sigma_s) - \sigma_s q_{n-1}\lambda_s}{(1 - \sigma_s)} = \frac{m_s\lambda_s(1 - \sigma_s) - \sigma_s q_{n-1}\lambda_s}{(1 - \sigma_s)} \tag{8}$$

where, $s = 1, 2, 3, \ldots$, $\kappa_s = \frac{(r_n)_{\lambda_s} - (r_n)_{\lambda_1}}{2(r_n)_{\lambda_s}}$, $\sigma_s = \left(\frac{r_1}{r_n}\right)^2_{\lambda_s}$, and i is the integer increment or decrement of the current interference order with respect to m_1. Using Eqs. 6 and 8, the following equation can be obtained:

$$B_{s1} = \frac{B_s}{B_1} = \frac{[(m_1 + i + \kappa_s)\lambda_s(1 - \sigma_s) - \sigma_s q_{n-1}\lambda_s](1 - \sigma_1)}{[m_1\lambda_1(1 - \sigma_1) - \sigma_1 q_{n-1}\lambda_1](1 - \sigma_s)} \tag{9}$$

The initial interference order in the object image can be expressed as follows:

$$m_1 = \frac{(i + \kappa_s)\lambda_s + q_{n-1}\left[B_{s1}\frac{\sigma_1}{(1-\sigma_1)}\lambda_1 - \frac{\sigma_s}{(1-\sigma_s)}\lambda_s\right]}{B_{s1}\lambda_1 - \lambda_s} \tag{10}$$

In this study, the wavelength of monochromatic light can be continuously varied by ± 0.05 nm. Therefore, the spectral dispersion coefficient of object birefringence is nearly equal unity for many objects $\left(B_{s1} = \frac{B_s}{B_1} \cong 1 \right)$. The initial interference order m_1 can be calculated from an approximate general formula as follows:

$$m_1 = \frac{(i + \kappa_s)\lambda_s + q_{n-1}\left[\frac{\sigma_1}{(1-\sigma_1)}\lambda_1 - \frac{\sigma_s}{(1-\sigma_s)}\lambda_s \right]}{\lambda_1 - \lambda_s} \quad \text{or}$$

$$m_1 = \frac{(i + \kappa_s)\lambda_s + q_{n-1}\left(\lambda_1 \left(\frac{r_1^2}{r_n^2 - r_1^2} \right)_{\lambda_1} - \lambda_s \left(\frac{r_1^2}{r_n^2 - r_1^2} \right)_{\lambda_s} \right)}{\lambda_1 - \lambda_s} \tag{11}$$

Using this formula, the initial interference order and the current interference orders in the highly oriented object image can be quickly and precisely calculated with computer-aided variable-wavelength multi-fringe optical Fourier transform technique. Also, it need not analyze the intensity of light at the interference pattern center to measure the initial interference order and apply the traditional multi-fringe technique to calculate the current interference orders in the highly oriented objects. The dispersion of the object birefringence can be calculated, using Eq. 8.

SETUP OF OPTICAL APPARATUS [8]

A standard polarizing microscope that produces the optical Fourier transforms (conoscopic observation) of highly oriented objects is shown in Figure 2a. The slit diaphragm D is located in the front focal plane of the condenser C, thus the polarized parallel beam is incident to the highly birefringent object. The optical Fourier transform is observed in the exit pupil of the microscope objective. The normal microscopical image $F^{\backslash}$ of the birefringent object can be observed in the image plane when the Bertrand lens (BL) is removed from the path of the light as shown in Figure 2b.

Using special software (Radial Vector Tracing Program), the interference pattern is analyzed automatically to measure the radii of the annular dark fringes as function of the light wavelength. A special program is prepared to determine the initial interference order, the current interference orders in the image of the object under study. Finally, the spectral dispersion curves of the highly oriented objects are calculated.
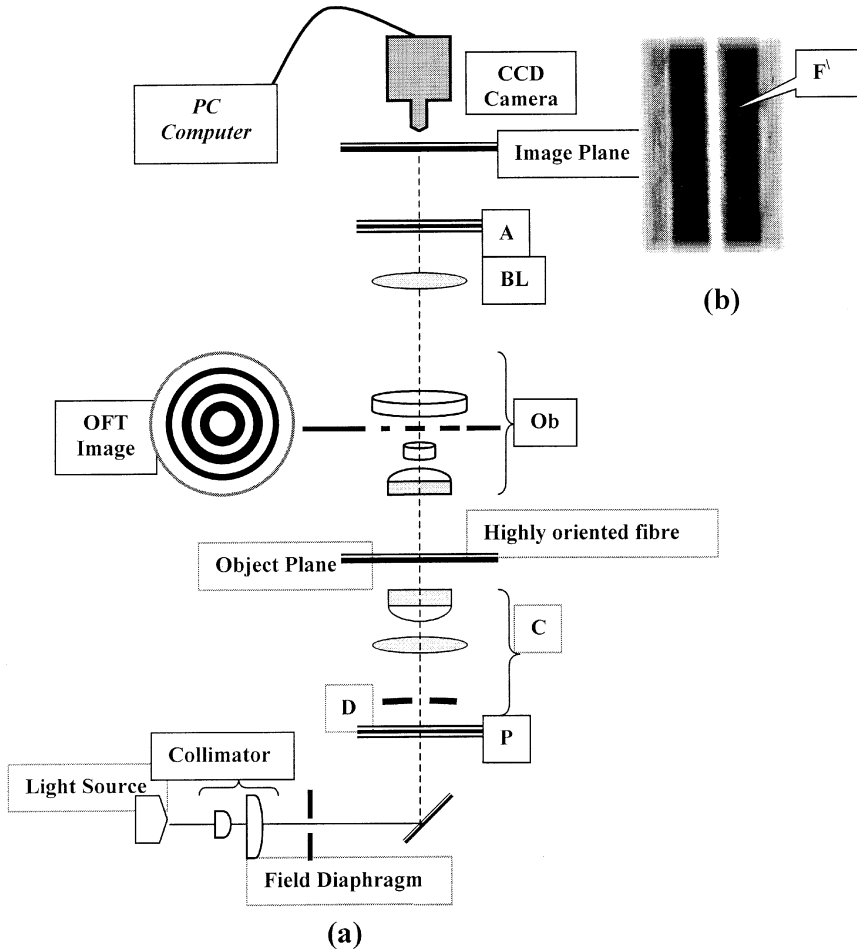
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FIGURE 2 Schematic diagram of the automatic computer-aided polarizing microscope system used for conoscopic observation and processing of the optical Fourier transforms patterns of birefringent objects.

EXPERIMENTAL RESULTS AND INTERPRETATION

The Pluta polarizing interference microscope is adapted to conoscopic observation (*Optical Fourier Transform technique*) as shown in Figure 2. The conoscopic mode of observation is a well-known method in polarized-light microscopy. It is based on illuminating the object under study with a wide-angle conical beam of light and consists in observing the interference patterns in the exit pupil (the back focal

plane) of a highly numerical aperture objective. The conoscopic interference pattern can be seen either by removing the microscope ocular or by maintaining the ocular and inserting the Bertrand lens *BL*.

A cylindrical highly birefringent PEEK fiber of diameter 27.52 μm is placed diagonally between two crossed polarizers as shown in Figure 2. The slit diaphagram (D) of the subcondenser (C) is oriented parallel to the fiber axis, as shown in Figure 3. An optical Fourier transform, which consists of two annular dark fringes and is simultaneously visible in the exit pupil of a microscope objective (objective magnification/numerical aperature: $40 \times /0.65$) as shown in Figure 4, occurs when a light wavelength of 350 nm is used. Their sizes and diameters depend on the wavelength λ of the monochromatic light used. A digital tunable liquid crystal filter is *used* to obtain the
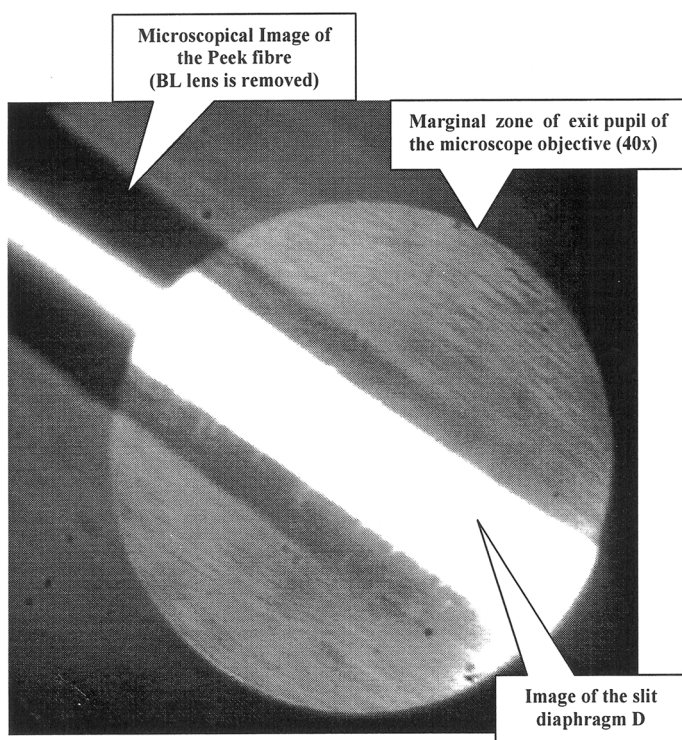

FIGURE 3 Microscopical images of birefringent PEEK fiber and the exit pupil of the microscope objective without and within the positive BL lens (Bertrand lens), respectively.
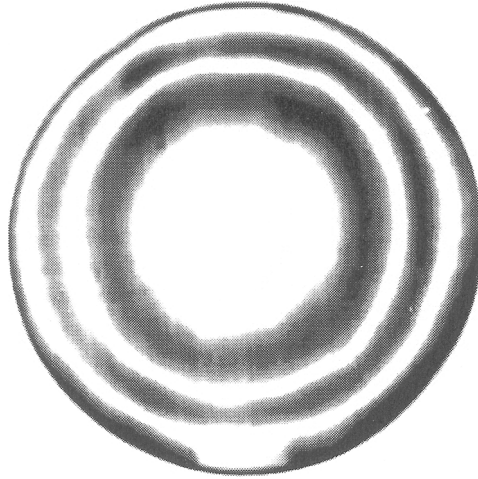
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FIGURE 4 Optical Fourier transform pattern produced by PEEK fiber in the back focal plane when the light wavelength $\lambda = 560$ nm is used.

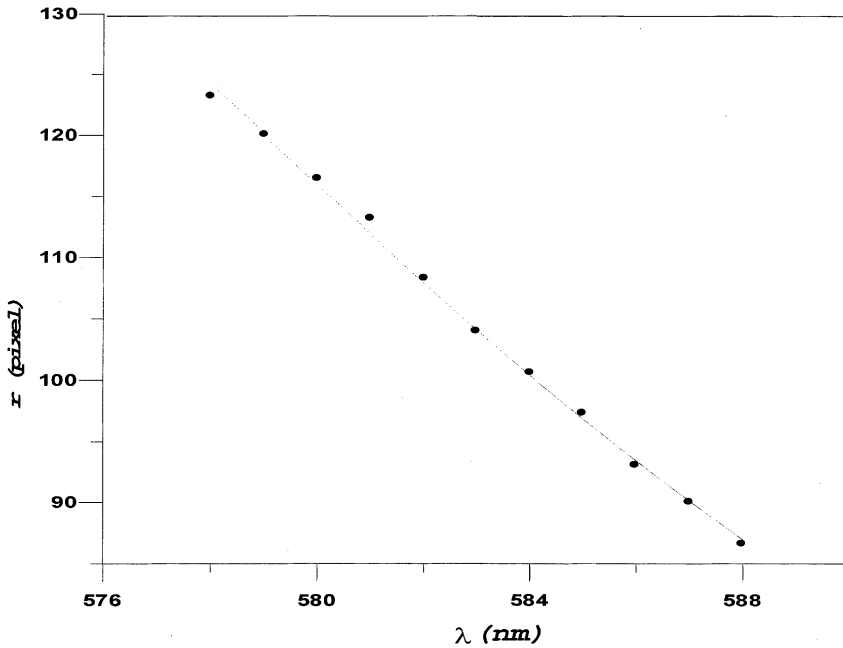**FIGURE 5** The relationship between the radius (r) of one annular dark fringe of PEEK fiber and the light wavelength (λ) for one interference sequence.

monochromatic light of continuously variable-wavelength in the visible spectrum. For each interference sequence of optical Fourier transform patterns the radii of the annular dark fringes increases when the light wavelength varied from a long to a short wavelength. Figure 5 displays the relation between the radius (in pixel) of one annular dark fringe and the light wavelength for one interference sequence. Having measured the radii $\{r_1(\lambda)$ and $r_2(\lambda)\}$ of only two neighbour annular dark fringes ($q_{n-1} = \pm 1$) over small range of the visible spectrum, the initial interference order in the PEEK fiber image can be determined using Eq. 11. The current interference order m_s ($= m_1 + 1 + \kappa_s$) is calculated. Applying Eq. 12, the special dispersion of PEEK fiber birefringence $B(\lambda)$ is determined as shown in Figure 6.
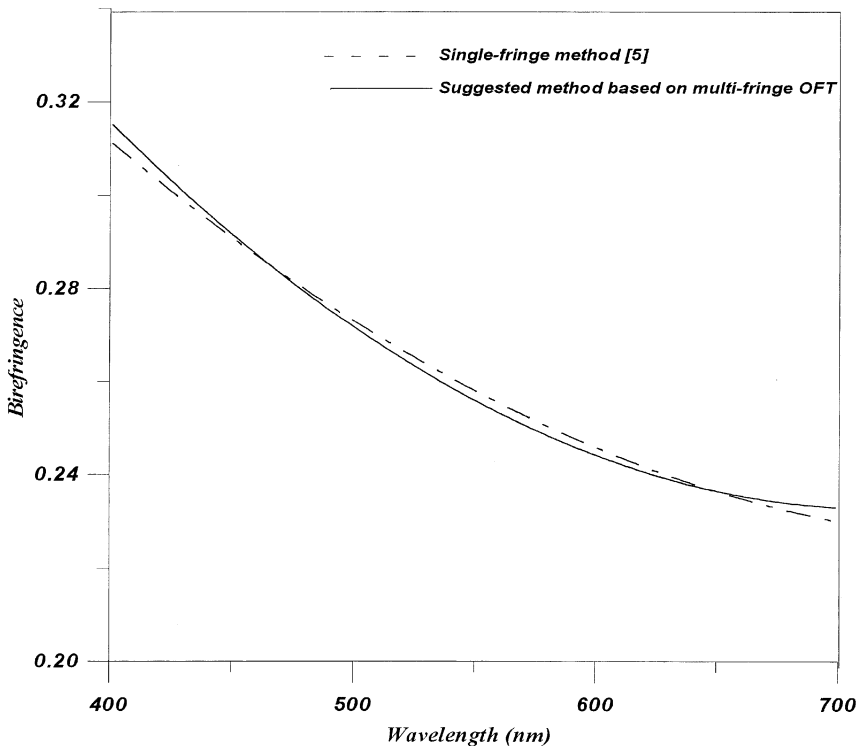

FIGURE 6 The spectral dispersion of PEEK fiber birefringence using the suggested interferometric method (solid line) and the method used by Sadik and Litwin [5] (broken line).

The modified single-fringe optical Fourier transform method [5] is applied to calculate $B(\lambda)$ of PEEK fiber for the sake of comparison, as shown in Figure 6. It is seen that the results obtained using both the suggested interferometric method and modified single-fringe optical Fourier transform approximately agree with each other.

In order to confirm the illustration of the current suggested interferometric method, the measurements have been performed on highly oriented Matrix fiber of thickness 22.9 microns. Figure 7 shows the optical Fourier transform pattern that consists of three annular dark fringes when light wavelength of 667 nm is used. The wavelength of monochromatic light can be varied from the long to short region in visible spectrum. The optical Fourier transform patterns are captured by CCD camera. The radii of the annular dark fringes of the interference pattern corresponding to its wavelength are automatically measured. Applying Eq. 11, the initial interference order is directly determined. Figure 8 demonstrates the comparison between the spectral dispersion of the Matrix fiber using the suggested interferometric method (solid line) and the method used by Sadik and Litwin [5] (dashed line). It is worth noting that the suggested interferometric method presented in this article can be used for the accurate measurement of the initial interference order, the current interference orders in the image of the highly oriented fibers.
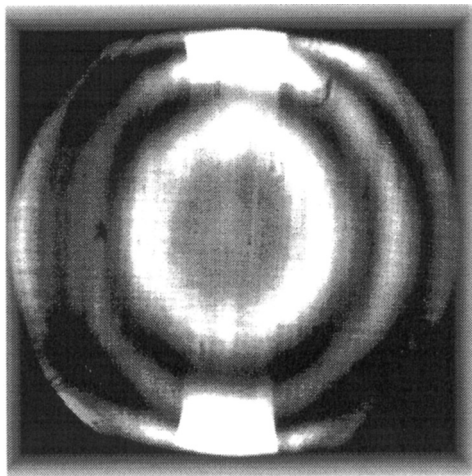

FIGURE 7 Optical Fourier transform pattern produced by Matrix fiber in the Fourier plane when the light wavelength $\lambda = 667$ nm is used.
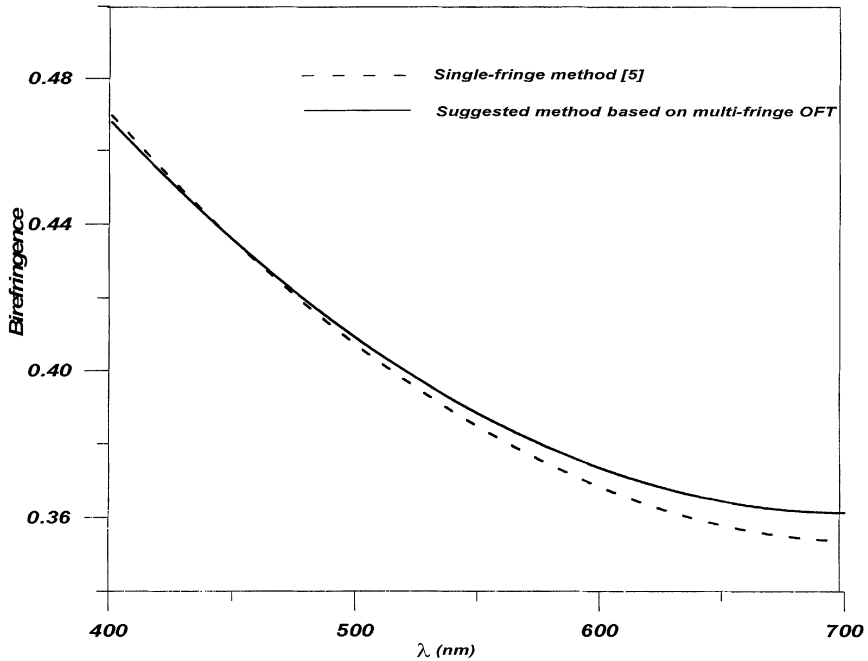
FIGURE 8 The spectral dispersion of Matrix fiber birefringence using the suggested interferometric method (solid line) and the method used by Sadik and Litwin [5] (broken line).

CONCLUSION

An interferometric method based on the multi-fringe optical Fourier transform technique is used to increase the accuracy of the direct measurement of initial interference order and the current interference orders in the image of the highly oriented object under investigation.

Using this method, the radius of the annular dark fringe is only the measuring optical parameter. Therefore, it need not measure the ordinary and extraordinary focal lengths and their difference for the highly oriented object as in the method used by Sadik and Litwin [5]. Also, the measuring error is decreased. The radii of the annular dark fringes are automatically measured over the range of the visible spectrum. The radius is measured in pixel. So it is not necessary to calibrate the optical system. Using Eq. 11, the initial interference order in the image of the highly oriented object is determined. Finally, the current interference orders and the spectral dispersion of object birefringence are determined.

Additionally, the tracing of the light intensity at the center of multi-fringe optical Fourier transforms patterns is ignored. This means that some time-consuming operations are removed. This proposed inter-ferometric method is quite simple in its procedures and gives good results.

REFERENCES

[1] D. Litwin and A. M. Sadik, JIAO OPTYKA, **No. 3**/**4**, 27−40 (1997) (in Polish).

[2] D. Litwin and A. M. Sadik, *Laser Interferometry IX*: *Applications*, in: Ryszard J. Pryputniewicz, Gordon M. Brown, Werner P. O. Juptner, Eds., *Proceedings of SPIE*, San Diego, USA, **3479**, 160−169 (1998).

[3] D. Litwin and A. M. Sadik, *Opt. Appl.* **28**, 139−154 (1998).

[4] D. Litwin and A. M. Sadik, *Molecular Crystals and Liquid Crystals* **353**, 485−498 (2000).

[5] A. M. Sadik and D. Litwin, *J. Opt. A: Pure Appl. Opt.* **4**, 135−139 (2002).

[6] M. Pluta, *Einladung und Program zur Tagung*, "Optik und Feinmechanik," der DgaO (Wetzlar: DGAO) **36** (1981).

[7] M. Pluta, *J. Modern Opt.* **34**, 1451−1470 (1987).

[8] M. Pluta, *Proceedings of SPIE*, San Diego, USA, **1603**, 566−576 (1991).